1. Introduction

The furniture that surrounds us was created using a variety of materials and forms. The vast majority of these materials came from non-recyclable natural sources. The question of whether we must produce every component of these furnitures caught my attention. In my opinion, even if they have a distinctive design, their function may still be served by using less material, which is a method that considers nature. Furniture that is fully functional and uses minimal materials can be designed in this way. How to quantify the applied force on furniture at each place is the key question. The spread of force on furniture is going to be understood using mechanical test simulation which is finite element method. The chair in my room that I've been using for the past five years was my choice for furniture; it has iron legs and a circular shape. I thought that its design might be improved to a point where it is less reliant on the natural world. Additionally, through supporting responsible consumption and production, the sustainable development goals may be achieved.

How does a chair leg's geometry (circle, triangle, square, pentagon, hexagon) impact its mechanical properties, with respect to the finite element approach?

2. Background Information

I have previously learned the specifics of applied forces on a three-dimensional object from my IB Mechanics physics classes. In this study, I want to look closely at a piece of furniture and how its geometry changes in response to force.

Everything is exposed to force in the system or region where it is used. This force is generated depending on the unique stress and strain values of each item. The structure of an object determines its performance, which is quantified by its stress and strain values (Chen et al., 2015). In this case, the area value is the cross-section area (A), which refers to the area generated when the item is cut into sections. Stress (σ) is the amount of force per area on an object. Here is how it works:

$$\sigma = \frac{F}{A} \tag{1}$$

Where,
σ = Stress value
F = Applied force
A = Cross-sectional area

Strain (ε) is a unit used to represent how much an item is moved when subjected to force. The end location is subtracted from the starting position and then dived to in order to calculate the value. The following is the formula for strain value:

$$\varepsilon = \frac{L - L_O}{L_0} = \frac{\Delta L}{L_0} \quad (2)$$

Where,
ε = Strain
L_0 = First position of the object
L = Final position of the object

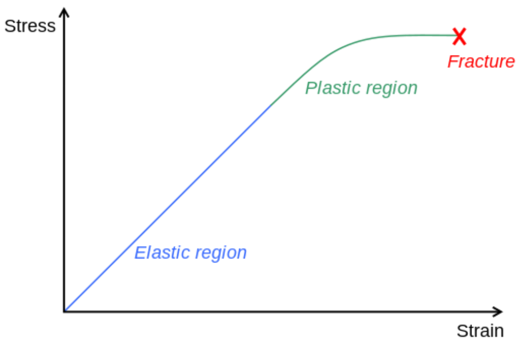
The values that will be computed in this research to comprehend an object's distinctive mechanical characteristics are described in the next section.

Elastic Modulus (Young's Modulus)

Deformation is the term for an item changing shape when a force exceeds a stress value. A brief deformation called elastic deformation occurs when force is applied. The material withstand to resist the applied force while maintaining its original form. The elastic modulus refers to the amount of elastic deformation on a force-applied item. As shown, Hook's Law, which states as follows, is employed here to determine the elastic deformation.

$$\sigma = E\varepsilon \quad (3)$$

Where,
σ = Stress value
E = Elastic modulus (Young's modulus)
ε = Strain



Graph 1 A stress and strain graph.

Since elastic deformation is not irreversible, the item will revert to its original shape as soon as the applied force is removed. In contrast to elastic deformation, when a force is applied, plastic deformation cannot return to its previous shape. Stress and strain values are not regulated during plastic deformation since the ratio between them is not at all calculated.

According to Graph 1, elastic and plastic deformation occur initially. The fracture, which causes an irreversible distortion, is the last point.

Resilience

The quantity of energy absorbed during elastic deformation is known as resilience. The resilience modulus equation is as follows:

$$u_r = \frac{{\sigma_y}^2}{2E} \tag{4}$$

Where,
u_r = Resilience modulus
σ_y = Yield strength
E = Elastic modulus

Yield strength value changes from material to material, and it's unit is Pascal.

Rigidity (Stiffness)
The definition of rigidity is a material's resistance to deformation. A stiff substance maintains its form even when applied with relatively strong forces. In contrast, a material that is readily deformed is referred to as flexible. Rigidity formula as follows:

$$k = E\frac{A}{L} \tag{5}$$

Where
k = Rigidity
E = Elastic modulus
A = Cross-section area
L = Displacement when force is applied on object

3. Hypothesis

The furniture is made using a variety of geometries. Each geometry has a unique form, thus when force is applied, the stress and strain values would vary. I anticipate that different chair leg geometries (circle, triangle, square, pentagon, hexagon) will exhibit distinct mechanical behavior. At this way a new approach for responsible manufacturing and consumption may be devised by taking into consideration the mechanical properties of each geometry.

4. Experimantal Design

Independent Variables

- Geometry of the object (circle, triangle, square, pentagon, hexagon)

Dependent Variable

- Elastic Modulus
- Resilience
- Rigidity
- Displacement

Controlled Variable

According to the chair in my room, controlled variables are chosen. About 500 mm^2 of the surface was in contact with the ground. The value for the chair's length was taken from

Neufert's Architect's Data, a reference work on the spatial needs of building design (Ernst Neufert et al., 2019).

- Ground contact surface (500.000 $\pm$0.001 mm^2)
- Length of the chair leg (440.000 $\pm$0.001 mm)
- The quantity of material used
- Applied mesh density
- Applied material (AISI 4340 Steel, normalized)
- The quantity of force applied (176.580 $\pm$0.001 N)
- The direction of force applied
- The size and the tolerance of mesh (12.000 $\pm$0.001 mm / 0.500 $\pm$0.001 mm)

Apparatus

To comprehend an object's mechanical properties, specialized laboratories are needed. Three-dimensional modeling and mechanical testing are both included in the computer program SolidWorks. Producing the desired design of metal object is not necessary in this way. Additionally, because it is a computer-based program, random and systematic errors are reduced in this way.

Using a computer-based experiment which is finite element method has the benefit of parametric results, which make it possible to compute desired values for each local point. Meshing, a technique that involves disassembling a three-dimensional object into tiny tetrahedrons, is used to complete the experiments. Tetrahedron intersection points are utilized in this way as parametric values. The name "finite element method" is comes from the determined parametric values (Desai et al., 2011). Topology optimization, which proposes a new design for a component in a mechanical system, also uses the finite element method (Sigmund & Maute, 2013).

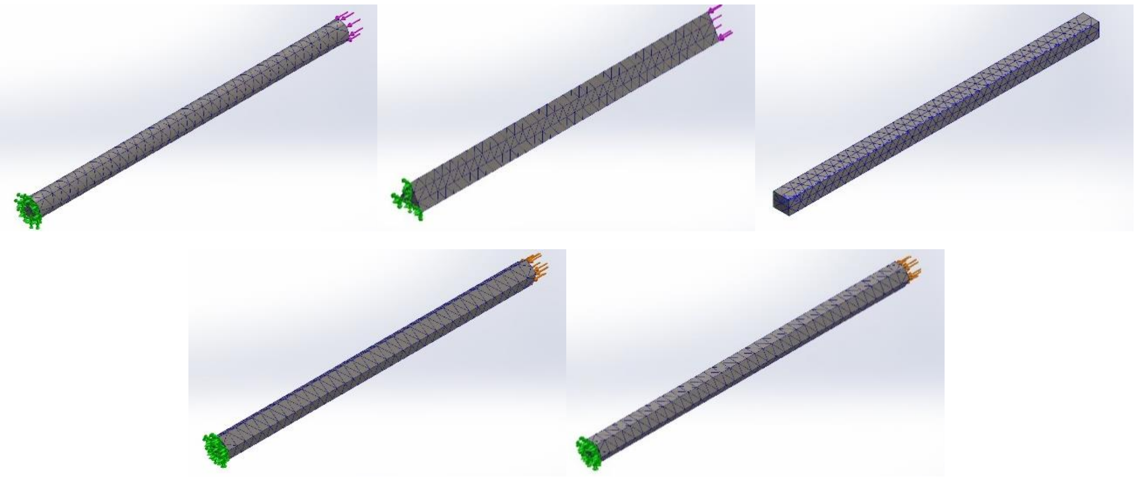


Figure 1 In orderly meshing process of circle, triangle, square pentagon and hexagon forms.

Experimental Procedure

1. In SolidWorks a circle, triangle, square, pentagon, and hexagon are generated each with a 500.000 mm^2 surface area.

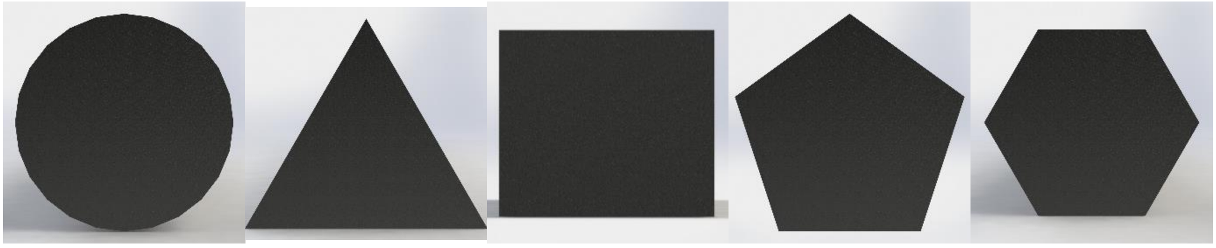


Figure 2 Circle, triangle, square, pentagon, and hexagon geometries all have the same surface area.

2. The shape of the form is produced by a 440.000 mm extruder.
3. The static study of finite element method is opened from the simulation page.
4. An external load linear 176.580 N force is applied to the surface opposite the ground contact surface, the ground contact surface is selected for fixed fixtures, and the material "AISI 4340 Steel, normalized" is selected.
5. The tolerance value is set to 0.500 mm, and the global mesh size is 12.000 mm.
6. Results for stress, strain, and displacement are obtained after each static analysis has been completed.
7. Steps 2, 3, 4, 5, and 6 are repeated for each shape.

Ethical and Enviromental Considerations

Because experiments are run by computer programs, only electrical energy is consumed. Since the employed program, SolidWorks, is not open source, it is impossible to be completely certain of the outcomes. This application is utilized since there is no alternative for it. Due to the application of sophisticated machinery and expensive materials, producing a metal with the correct shape is an expensive process. Utilizing simulation eliminates the requirement to produce samples (Kakani & Kakani, 2004).

Assumptions

With reference to my mass, the force that would be applied to each chair leg is calculated. According to Newton's second law, my mass must be multiplied by the acceleration of free fall to determine how much I weigh in Newtons.

$$F = mg \tag{6}$$

Where,

F = Weight (N)

m = Mass of mine (72.00 kg)

g = Free fall acceleration (9.81 ms^{-2})

$$F = 72.00\ kg \times 9.81\ ms^{-2} \quad (7)$$

The results show that I am 706.3 N in weight. In order to calculate the force acting on each chair leg, this number is divided into four equal parts. Since force does not equally distribute among chair legs, this is an assumption.

In order to calculate the applied force, a linear force is used, which is not always practical in real world applications.

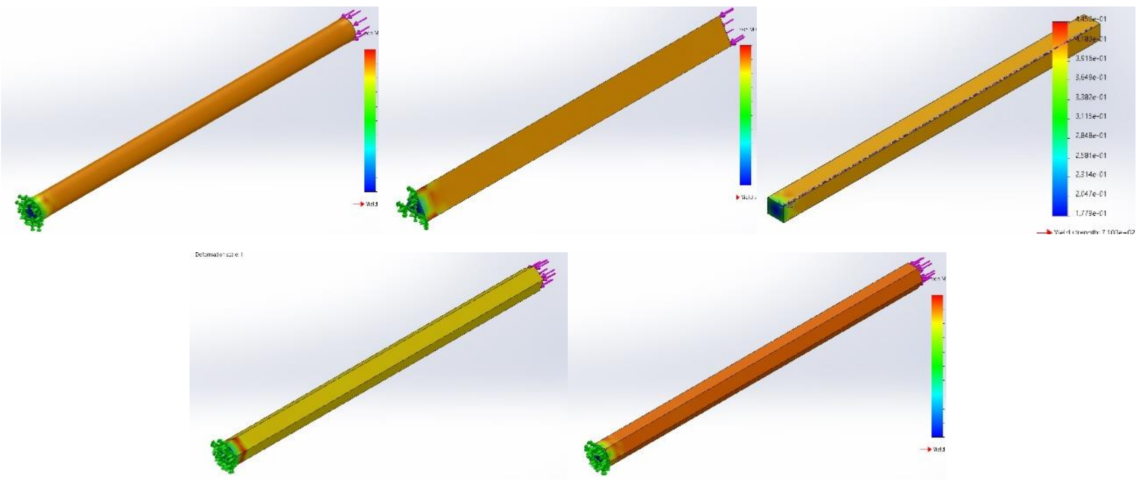
The manufacture of chairs makes use of a wide variety of materials. The mechanical properties of normalized AISI 4340 steel will be tested in this investigation which has 470 Mpa yield strength (AZoM, 2012). Since the data would be correlated with material attributes, it could be used to understand the mechanical properties of various materials.

5. Analysis of Data

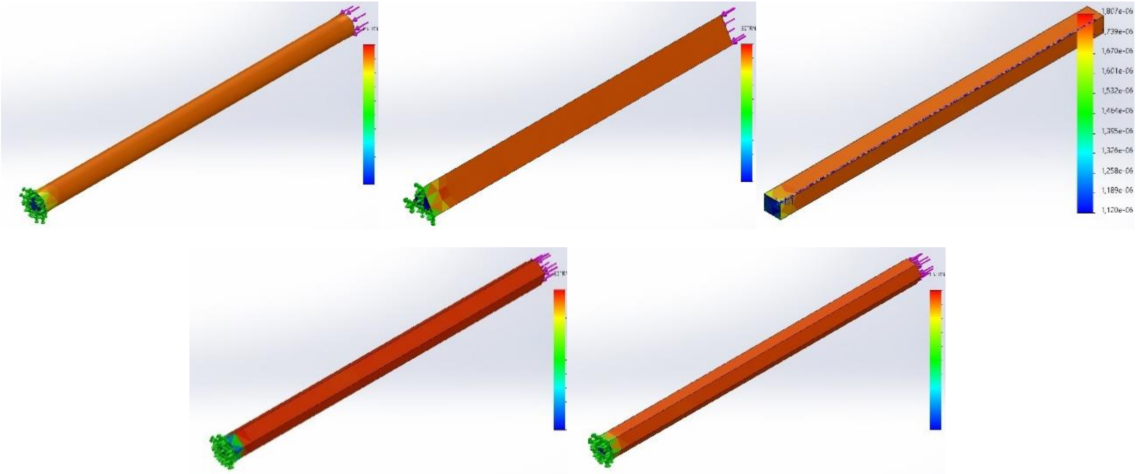
Qualitative and quantitative data are separated into two categories at the conclusion of the simulation tests. Data on stress, strain, and displacement values as well as their visual stimuli were obtained.

Qualitative Data

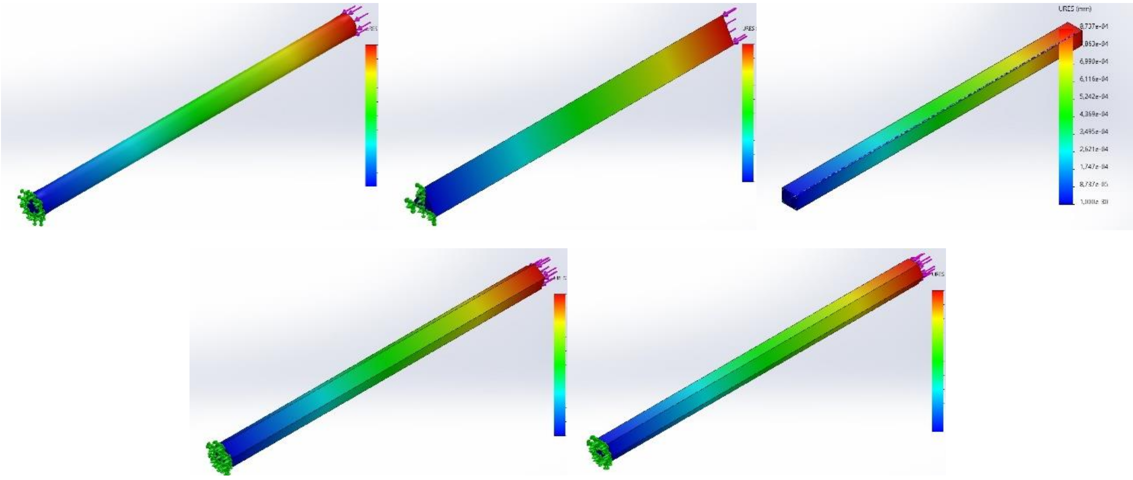
Views of each object used in the experiment served as the qualitative data for this study. Each geometry responded differently to each value. The obtained data are presented in this instance as visual information, and these values demonstrate the parametric values, which are local values. Values for parametric stress, strain, and displacement are displayed using colors ranging from blue (minimum) to red (maximum).



Graph 2 Stress values are illustrated parametrically on each form, respectively circle, triangle, square, pentagon, hexagon.



Graph 3 Strain values are illustrated parametrically on each form, respectively circle, triangle, square, pentagon, hexagon.



Graph 4 Displacement values are illustrated parametrically on each form, respectively circle, triangle, square, pentagon, hexagon.

Quantitative Data

Table 1 The obtained stress, strain and displacement values for each geometry.

Geometry	Stress Value $(\pm 1 \times 10^{-4}\, N/mm^2)$	Strain Value $(\pm 1 \times 10^{-9})$	Displacement Value $(\pm 1 \times 10^{-7} mm)$
Circle	3.517×10^{-1}	1.517×10^{-6}	3.784×10^{-4}
Triangle	3.519×10^{-1}	1.513×10^{-6}	3.774×10^{-4}
Square	4.062×10^{-1}	1.748×10^{-6}	4.361×10^{-4}
Pentagon	3.511×10^{-1}	1.507×10^{-6}	3.744×10^{-4}
Hexagon	3.513×10^{-1}	1.512×10^{-6}	3.776×10^{-4}

Processed Data

The obtained raw data is used in Table 1 to generate meaningful values for critical parameters that determine an object's mechanical properties. Elastic modulus, resilience, and rigidity are these. Since the experiment was done using a simulation program, there is less uncertainty about the results. The generated qualitative and quantitative results remained unchanged even after running the simulation again. Even if there were only four decimal places available for the expression of quantitative data, this also implies that the next decimal place is uncertain. An uncertainty value using the tenth system is added to determine the uncertainty after the provided or set number's last decimal point. For the for the length stress value $\pm 1 \times 10^{-4}\ \mathrm{N/mm^2}$, for the strain value $\pm 1 \times 10^{-9}$and for the displacement value $\pm\ 1 \times 10^{-7}$mm is used.

1. Sample Calculation for Circle: The values from the first and third rows of Table 1 are entered into the following equation, which is derived from Equation (3), to calculate **elastic modulus and related uncertainty**.

Calculation of Elastic Modulus

$$\mathrm{E} = \frac{\varepsilon}{\sigma}$$

$$= \frac{1.517 \times 10^{-6}}{3.517 \times 10^{-1}}$$

$$= 4.313 \times 10^{-1} Pa$$

Related Uncertainty

$$\frac{\Delta \mathrm{E}}{E} = \frac{\Delta \varepsilon}{\varepsilon} + \frac{\Delta \sigma}{\sigma}$$

$$= \left(\frac{1 \times 10^{-9}}{1.517 \times 10^{-6}} + \frac{1 \times 10^{-4}}{3.517 \times 10^{-1}} \right) \times 100$$

$$= (9.435 \times 10^{-4}) \times 100$$

$$= 0.09\%$$

2. Sample Calculation for Circle: To calculate the resilience modulus Equation (4) is used. In this study the used material is normalized AISI 4340 Steel and it's yield strength in literature is 470 MPa. For E value the calculated values are used in the first sample calculation. Using the following equation **resilience modulus and and related uncertainty** are determined for each geometry.

Calculation of Resilience Modulus

$$u_r = \frac{(470 \times 10^6)^2}{2E}$$

$$= \frac{(470 \times 10^6)^2}{4.313 \times 10^{-1}}$$

$$= 4.764 \times 10^{16}\ Jm^{-3}$$

Related Uncertainty

$$\frac{\Delta u_r}{u_r} = \frac{\Delta E}{E}$$

Since the uncertainty of the circle's elastic modulus is 0.09 percent, the same holds true for its resilience modulus, because the resilience modulus was calculated without the need of any extra measurement.

$$= 0.09\%$$

3. Sample Calculation for Circle: Equation (5) is used to determine the rigidity of various geometries. For each geometry in equation A, which is determined as a controlled variable with a value of 500.000 $\pm$0.001 mm^2, L is acquired from Table 1's third row, and E is derived from the first sample calculation. The **stiffness value and related uncertainty** for each shape is calculated using the following equation.

Calculation of Resilience Modulus

$$k = E\frac{500.000}{L}$$

$$= 4.313 \times 10^{-1} \times \frac{500.000}{3.784 \times 10^{-4}}$$

$$= 5.699 \times 10^{1}\ N/mm$$

Related Uncertainty

$$\frac{\Delta \mathrm{k}}{\mathrm{k}} = \frac{\Delta \mathrm{L}}{L} + \frac{\Delta \mathrm{E}}{E}$$

$$= \left(\frac{0.001}{500.000} + \frac{1 \times 10^{-7}}{3.784 \times 10^{-4}} \right) \times 100 + 0.09$$

$$= (2.64 \times 10^{-2}) \times 100 + 0.09$$

$$= 2.73\%$$

Using Table 1 and sample calculations, the Table 2 is generated.

Table 2 The elastic modulus, resilience modulus, and displacement mechanical characteristics of geometries are shown.

Geometry	Elastic Modulus ($\pm 0.09\%\ Pa$)	Resilience Modulus ($\pm 0.09\%\ Jm^{-3}$)	Rigidity ($\pm 2.73\%\ N/mm$)	Displacement Value ($\pm 1 \times 10^{-7}\ mm$)
Circle	4.313×10^{-1}	4.764×10^{16}	5.699×10^{1}	3.784×10^{-4}
Triangle	4.300×10^{-1}	4.749×10^{16}	5.696×10^{1}	3.774×10^{-4}
Square	4.303×10^{-1}	4.753×10^{16}	4.934×10^{1}	4.361×10^{-4}
Pentagon	4.292×10^{-1}	4.741×10^{16}	5.732×10^{1}	3.744×10^{-4}
Hexagon	4.304×10^{-1}	4.754×10^{16}	5.699×10^{1}	3.776×10^{-4}

6. Conclusion

In this research, the relationship between a shape's geometry and its mechanical characteristics is explored. Four parameters are chosen to compute in order to compare the attributes of various geometries. These are the displacement, rigidity, rigidity modulus, and elastic modulus. The creation of items in a 3D design application is followed by simulation tests which is finite element method. Qualitative visual graphs of stress, strain, and displacement are produced as a consequence of simulation testing, and quantitative data average values of stress, strain, and displacement are derived. To acquire the necessary parameters, the quantitative values are processed. Finally, unique values are found for each geometry and are given in Table 2.

The quantitative findings demonstrate that each geometry has unique characteristics, yet these values are not significantly different from others. The following is the order for elastic modulus:

$$Circle > Triangle > Square > Hexagon > Pentagon \tag{8}$$

The following is the order for resilience modulus:

$$Circle > Hexagon > Square > Triangle > Pentagon \tag{9}$$

The following is the order for rigidity value:

$$Pentagon > Circle = Hexagon > Triangle > Square \tag{10}$$

The following is the order for displacement value:

$$Square > Circle > Hexagon > Triangle > Pentagon \tag{11}$$

The qualitative findings indicate that the homogeneity of each tested value differs for each shape when parametrically created. Stress graphs demonstrate that the distribution of the applied force is either homogeneous or heterogeneous depending on the geometry. Because the force is not concentrated at one location, which may be dangerous, the dispersal of force was intended to be homogeneous in order to have a more stable bar. The geometries below are arranged from homogeneous to heterogeneous:

$$Hexagon > Circle > Triangle = Square > Pentagon \tag{12}$$

Strain graphs demonstrate that there are homogeneous or heterogeneous strain values for each local region of the bars, just as there are in stress graphs. For any geometry, there aren't any really distinguishing features amongst the bars. The displacement graphs also demonstrate that there aren't many differences between the various geometries. The parametric values in all graphs are not significantly different from one another. Where the fixed geometry is used, the stress and stress values are concentrated there. The side of the bar where force is applied experienced higher variation in the displacement value due to parametric factors.

Table 3 The used geometries and their perimeter values.

Geometry	Perimeter (± 0.001 mm)
Circle	79.266
Triangle	101.943
Square	89.443
Pentagon	85.235
Hexagon	83.238

Since utilized each geometry has differenet perimeter, possible relation between perimeter and mechanical property is decided to investigate. As Table 5 demonstrates when corner number increased to infinity perimeter decreases. This trend shows no relation between obtained results. So at this way it is proved that the distinctive feature of each geometry which are the number of corner and perimeter have no relation between the geometries' mechanical property.

As a result of the design process, values were obtained that may be used to encourage more responsible production and consumption in the globe (United Nations, 2021). In this work, various geometric options for a chair leg design are studied. Bars are utilized in the majority of mechanical systems, not merely for the reason mentioned above. In response to the research question, it is possible to achieve responsible production and consumption by better understanding the mechanical characteristics of the geometries. Natural science approach may be used to assist global sustainable development, which directly benefits people. Utilizing the outcomes and taking into account the pressures and anticipated reactions on the bar in accordance with the intended usage, Here are the characteristics of each geometry and usage suggestions:

Circle: The ideal geometry to make a bar is a circle. Being in second position for stiffness and having the highest value for elastic and resilience modulus makes this geometry more durable than others. Since it has a larger displacement value than the others, it is possible to see a change from its original position when force is applied. Circle geometry would be the optimum option in a mechanical system when the location of the bar when force is applied is unimportant. Since the circle's shape in Graph 1 does not have completely homogen stress values, the area where stress is concentrated may deform.

Triangle: The obtained elastic and resilience modulus for triangle geometry is similar to that for circles. Triangle placed second in stiffness among other geometries, which gives it strong force resistance. Triangle's displacement value, which is the lowest among other geometries, is its distinguishing feature. In a mechanical system when this quality is crucial, triangular geometry would be the optimum choice since when a force is applied, the change in position would be minimal. Although the distribution is not entirely homogeneous, as seen in Graph 1, it is in good condition when compared to others.

Square: In terms of elastic and resilience modulus, square shape comes in second to other geometries. This geometry suffers from lower than average stiffness and higher than average displacement values compared to other geometries. As a result, it cannot be used in mechanical systems that are intended to be more stable since it is not very resistant to deformation when force is applied and there is a significant difference between the beginning and final positions. Graph 1 demonstrates that when force is applied, local spots on the bar have little variation in the stress value.

Pentagon: Even though the pentagon is the most rigid shape, it has a low resilience and elastic modulus value, making it resistant to deformation but not elastic. It implies that breaking the bar would happen quickly. Because rigidity in and of itself is not advantageous. According to Graph 1, which demonstrates the significant differences between nearby locations, the strongly concentrated red point would immediately break under stress.

Hexagon: Hexagon geometry always has average values for elastic modulus, resilience modulus, stiffness, and displacement. Given that not all bars are made with the objective in mind, it can increase the bar's use. The geometry would be more valuable in some situations than others, but overall, it is stable. Graph 1 further demonstrates that the hexagon has the least variation between local sites, making it the most homogeneous stress-spread shape when force is applied.

7. Evaluation

In this study, the mechanical characteristics of various geometries are examined. Physical laboratories and computer-based simulations are the two main methods used to test a metal's mechanical properties. In my case, making five bars with various geometry was not feasible. Furthermore, experiments are run in simulations to achieve better findings with minimum uncertainty. The identical experimental setting produced the same results in simulations, which do not take random mistakes into account.

As stated under the heading "Processed Data," the simulation does not provide an uncertainty value for the results; nonetheless, as the results are provided to four decimal places, the fifth decimal place is also assumed to be uncertain. For each outcome, an uncertainty value is calculated in order to address this situation. Uncertainty value is determined using a sample calculation for the required values. Calculated values include elastic modulus of ±0.09%, resilience modulus of ±0.09%, stiffness of ±2.73%, and displacement value of $\pm 1\times10^{-7}$.

Computer-based simulations lessen systemic mistakes since there are no outside influences that might affect the outcomes. Simulations are repeated on the same computer and an another computer to confirm the system, but the outcomes are unaltered.

Further Studies

The parametric stress spread values are displayed in Graph 1. This graph may be used to develop a new design that strengthens the geometry. For instance, in circular geometry, the stress is concentrated at a single point, making the geometry unstable. However, if a new design is developed with the location of the stress concentration in mind, this significant discrepancy between the local spots would be reduced. Only five alternative geometries are employed as independent variables in this inquiry, however further geometries might expand the literature.

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