Beamline for Schools

The Accuracy of Highland's Approximation of Multiple Scattering for Muon Beams in T9

Team Nabla

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Contents

1 Introduction

We are a group of juniors, from several high schools, who have come together to explore the magical realm of subatomic particles. In the process, every drop of sweat, every sleepless night, and every eraser litter empowered us with mounting eagerness to get a step closer to the final product. We questioned, investigated, and now are ready to test our ideas at CERN. Working with particle accelerators at CERN will be a fascinating experience since we will be able to perceive our theoretical derivations actualized. We consider it an honor to prepare a proposal in physics, to share knowledge, and to create a difference. In the same manner that Einstein and Curie left key findings to future generations, we intend to do the same.

As a team, we preferred to study a type of particle that is not thoroughly investigated. Considering the particles that secondary beams provide, the most suitable option was studying muons since pions' and kaons' lifetimes are too short to study at T9. Considering the environment and resources that are provided for a BL4S experiment, because only a target-based experiment is possible, we decided to concentrate on the scattering of muons with a target. After hours of research, we have found that using Highland's approximation of multiple scattering, would be suitable as we are dealing with high-energy particles that need a thicker target to scatter.

2 Experiment

2.1 Aim

We aim to see to what extent Highland's approximation of multiple scattering is accurate for Muon beams in T9.

2.2 Theoretical Background

There are several approximations used for the particle scattering for the target thickness. We employ Highland's approximation since we use thick targets for our muon beam.[i]

Highland approximation:

$$\theta_0 = \frac{13.6}{\beta \cdot c \cdot p} \cdot z \cdot \sqrt{\frac{x}{X_0}} \cdot \left[1 + \left(0.038 \cdot \ln\left(\frac{x}{X_0}\right)\right)\right]$$ [ii]

where:

$\theta =$ the solid angle into which is concentrated the 98% of the beam after a thickness X of material[iii]

$\theta_0 = \frac{\theta}{\sqrt{2}}$

$c =$ the speed of light

$\beta = \frac{v}{c}$ where v is the velocity of the particle

$p =$ momentum

$z =$ charge

$x =$ thickness of material

$X_0 =$ radiation length of target

[i] https://gray.mgh.harvard.edu/attachments/article/337/Techniques%20of%20Proton%20Radiotherapy%20(06)%20Multiple%20Scattering.pdf

[ii] https://meroli.web.cern.ch/lecture_multiple_scattering.html

[iii] https://meroli.web.cern.ch/lecture_multiple_scattering.html

When a charged particle passes through a slice of material with thickness x, it ionizes the atoms it encounters. The energy loss E by unit length, dE/dx, is crucial in the detection of stable charged particles because it is dependent on both particle properties as well as material properties. We will employ the Bethe-Bloch equation to examine the energy loss of the muon beam for the sake of our proposal.[iv]

The Bethe-Bloch equation:

$$-\left\langle \frac{dE}{dx} \right\rangle = \frac{4\pi}{m_\mu c^2} \cdot \frac{nz^2}{\beta^2} \cdot \left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 \cdot \left[\ln\left(\frac{2m_\mu c^2\beta^2}{I\cdot(1-\beta^2)}\right) - \beta^2\right]$$ [v]

where:

$$\begin{aligned}
c &= \text{ the speed of light}\\
\beta &= \frac{v}{c} \text{ where } v \text{ is the speed of the particle}\\
E &= \text{ energy of the particle}\\
x &= \text{ thickness of the material}\\
z &= \text{ the charge of the particle}\\
e &= \text{ fundamental charge}\\
m_\mu &= \text{ muon's rest mass}\\
n &= \text{ volume density of electron in the target material}\\
I &= \text{ ionization constant, which describes how easy it is to ionize its atoms}
\end{aligned}$$

Bethe-Bloch equation is needed to identify the muon beam after scattering from the thick target. The identification will be done by:

a) Subtracting energy loss from total energy that is preselected at the collimator to find the scattered particles' momentum, assuming that at high energies, momentum equals kinetic energy.

b) Determining the velocity of the scattered particles by using detectors.

c) Using the relation between momentum, velocity and mass to find the mass of the scattered particles, hence identifying muon.

The relation that will be used is as follows:

$$p = \frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma m_0 v$$ [vi]

where:

$$\begin{aligned}
m_0 &= \text{ initial mass}\\
v &= \text{ velocity}\\
p &= \text{ momentum}\\
c &= \text{ speed of light}\\
\gamma &= \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}
\end{aligned}$$

[iv] https://www.youtube.com/watch?v=eYFI_jKR6TQ
[v] https://tr.wikipedia.org/wiki/Bethe_form%C3%BCl%C3%BC
[vi] https://youtu.be/xkjEmUGNTsg

2.3 Method

The positive beam is chosen for this experiment because there are more pions and kaons in the positive beam than in the negative beam. The chosen momentum is 6 GeV because we want the pion and kaon to be most concentrated, which can generate a muon beam as intense as possible. But we don't want the positron and the proton to be much either, since they will be irrelevant.

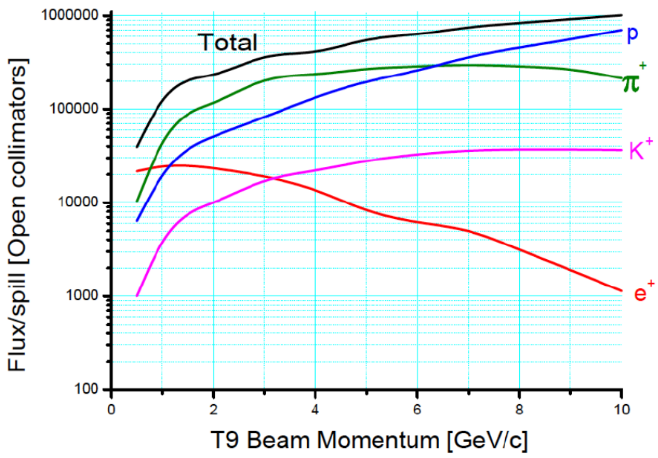

Figure 1: The estimated maximum flux in the positive beam[vii]

Cherenkov detectors will be used to identify the particles that travel to the experimental area. The experimental setup is as follows:

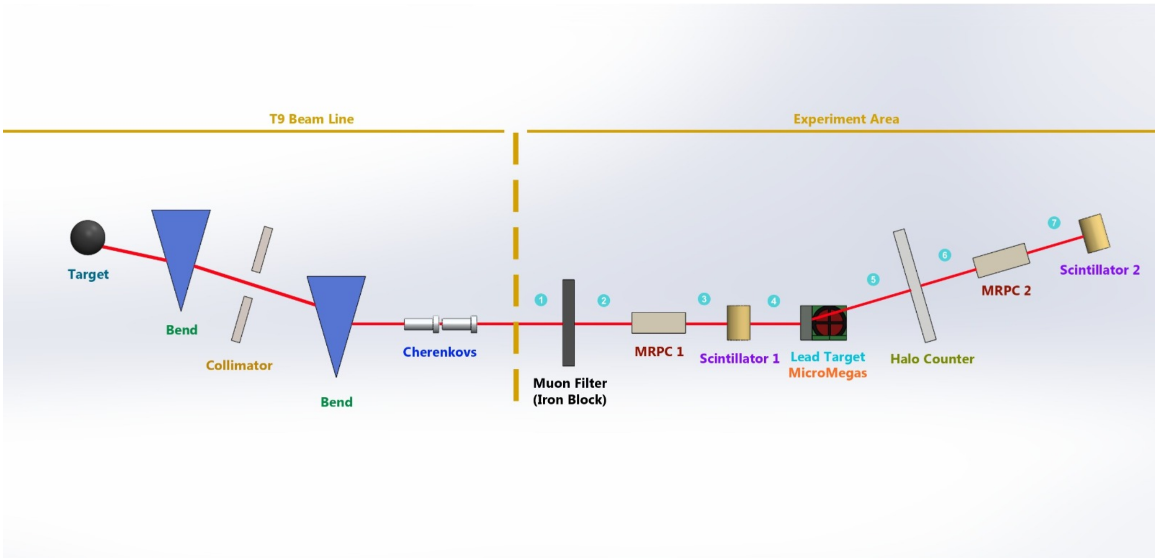

Figure 2: The experimental setup

1) The beam goes into a muon filter (an iron block) and particles other than muons are separated.

2) MRPC1 measures the velocity of the beam to identify if the beam is a muon beam or not.

3) SC1 is placed after the MRPC1 to count the muon particles.

4) The beam goes through a thick lead target. Using Highland's Multiple Scattering Approximation, we predict the angle at which 98% of the muon beam scatters. This angle will be later checked by the 2 MicroMegas' that are placed around the target.

[vii] http://beamlineforschools.cern/sites/default/files/Beams_Detectors_BL4S2022_1.pdf

5) The beam moves through a Halo Counter that is placed in the assumed direction of the muon beam which multiple scattering theory predicts. The Halo Counter will be set so that the particles around the halo counter are stopped. This will only allow the 98% part of the beam to pass. Since some variables lack values, this angle will be calculated in CERN, when the velocity is measured. The equation for multiple scattering with the other components' values is given by:

$$\theta_0 = \frac{13.6}{\beta \cdot c \cdot p} \cdot z \cdot \sqrt{\frac{x}{X_0}} \cdot [1 + (0.038 \cdot \ln(\frac{x}{X_0}))]$$ [viii]

Where:

$$\begin{aligned}
\theta_0 &= \frac{\theta}{\sqrt{2}} \\
c &= 3.0 \cdot 10^8 \text{ m/s} \\
\beta &= \text{ Will be calculated with MRPC since } v \text{ is required} \\
p &= 6 \text{ GeV} \\
z &= +1 \\
x &= 4 \text{ cm} \\
X_0 &= 6.3688\ g/cm^2
\end{aligned}$$ [ix]

6) The velocity of the incoming beam will be calculated through the use of MRPC2. This velocity value and the remaining energy calculated from the Bethe-Bloch equation will be used to identify if the beam is a muon beam or not. Again, we lack the velocity value; so the formula will be calculated in CERN. Bethe-Bloch equation with the other components' values is given by:

$$-\left\langle \frac{dE}{dx} \right\rangle = \frac{4\pi}{m_\mu c^2} \cdot \frac{nz^2}{\beta^2} \cdot \left(\frac{e^2}{4\pi\varepsilon_0}\right)^2 \cdot \left[\ln\left(\frac{2m_\mu c^2\beta^2}{I \cdot (1-\beta^2)}\right) - \beta^2\right]$$ [x]

Where:

$$\begin{aligned}
c &= 3.0 \cdot 10^8 \text{ m/s} \\
\beta &= \text{ Will be calculated with MRPC since } v \text{ is required} \\
E &= 6 \text{ GeV} \\
x &= 4 \text{ cm} \\
z &= +1 \\
e &= 1.6 \cdot 10^{-19} \text{ C} \\
m_\mu &= 105.7 \cdot 10^6\ eV/c \\
n &= \frac{N_a \cdot Z \cdot \rho}{A \cdot M_u} = 8.98\ g/cm \\
I &= 10 \text{ eV} \cdot 82 = 820 \text{ eV}
\end{aligned}$$

7) The beam goes through SC2 to record how many particles are scattered. This value will be compared with the value given at SC1 to see if 98% of the beam is really scattered at the angle that Highland's approximation predicts.[xi]

Given the formulas, we selected a thickness of 4 cm since we predict that it will provide an angle that can be studied more effectively. Also, we considered the energy loss that will be calculated from the Bethe-Bloch equation. The energy loss of muon particles should not be excessive so that the particles can exit the target.

[viii] https://meroli.web.cern.ch/lecture_multiple_scattering.html

[ix] Yung-Su Tsai, Pair production and bremsstrahlung of charged leptons, Reviews of Modern Physics, vol. 46, no. 815, 1974

[x] https://tr.wikipedia.org/wiki/Bethe_form%C3%BCl%C3%BC

[xi] https://meroli.web.cern.ch/lecture_multiple_scattering.html

3 What do we hope to take away from the experience?

BL4S competition presents us with a once-in-a-lifetime opportunity for high school students, we would like to expand our horizons by conducting our experiment with the sincere guidance of scientists at CERN. Before we lead the way for more students to get involved in physics, we want to confirm our high aptitude for physics and the scientific world. So, we hope to take competency and experience and turn them into a step that we can use for achieving another goal which is being a beacon of hope for our country's young scientists who underestimate themselves.

4 Acknowledgements

We'd like to express our gratitude to our teachers, Ms. Oskay and Mr. Çelik, for their help during the process. We would like to dedicate this work to our beloved friend, Ceren İnci Çolak, who passed away while we were working on this proposal. We wish to make her live eternally within here.

5 Bibliography

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5) https://youtu.be/xkjEmUGNTsg

6) http://beamlineforschools.cern/sites/default/files/Beams_Detectors_BL4S2022_1.pdf

7) http://beamlineforschools.cern/sites/default/files/Experiment_example_0.pdf

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